Tensor-based Feedback Control for Local Structured High-dimensional Streaming Data under Limited Control Capability

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Problem Statement

Utilize **high-dimensional (HD) streaming data** to perform **feedback control** to improve the product quality

Challenge I: Local Structure

- Local correlations within the system response
- Local patterns within the spatial impact of location-sensitive control variables

Challenge II: Limited Control Resources

- Insufficient funds to implement full-scale system control
- Lack of skilled manpower to execute the control actions
- Extreme environmental conditions limit the types of control resources that can be employed

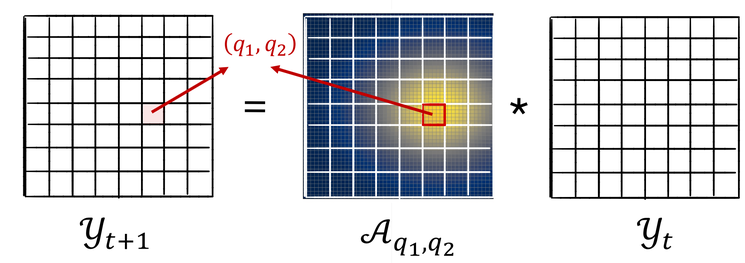
Objective Design an innovative tensor-based feedback control framework tailored for local structured high-dimensional streaming data under limited control resources

HD Gaussian Product Kernel

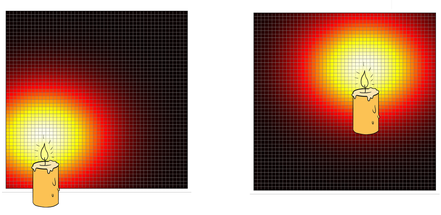
$$\mathcal{K}_{\boldsymbol{c},\boldsymbol{\sigma}}(\mathcal{D}_1,\dots,\mathcal{D}_l,\dots,\mathcal{D}_L;\boldsymbol{c},\boldsymbol{\sigma})=\prod_{l=1}^{L}\exp\left(-\frac{1}{2}(\mathcal{D}_l-c_l)^2/\sigma_l^2\right)$$

- center: $\boldsymbol{c}=(c_1,\dots c_L)$; bandwidth: $\boldsymbol{\sigma}=(\sigma_1,\dots,\sigma_L)$
- index tensor: $\mathcal{D}_l=1_{Q_1}\otimes\dots\otimes 1_{Q_{l-1}}\otimes D_l\otimes 1_{Q_{l+1}}\otimes\dots\otimes 1_{Q_L}$
  - $D_l\in\mathbb{R}^{Q_l}$ → indices along the l-th dimension
  - $\otimes$ → the outer product operation
  - $1_{Q_{l+1}}$ → a vector of ones with length Q_l along the l-th dimension

Local Structures in $\mathcal{A}$ and $\mathcal{B}$



$\mathcal{A}_{q_1,\dots,q_L}$ spatial local correlation → $(q_1,\dots,q_L)$-th entry in $\mathcal{Y}_{t+1}$



$\mathcal{B}_p$ unit control impact → p-th location-sensitive control variable

HD Gaussian Product Kernel → $\mathcal{A}$ and $\mathcal{B}$

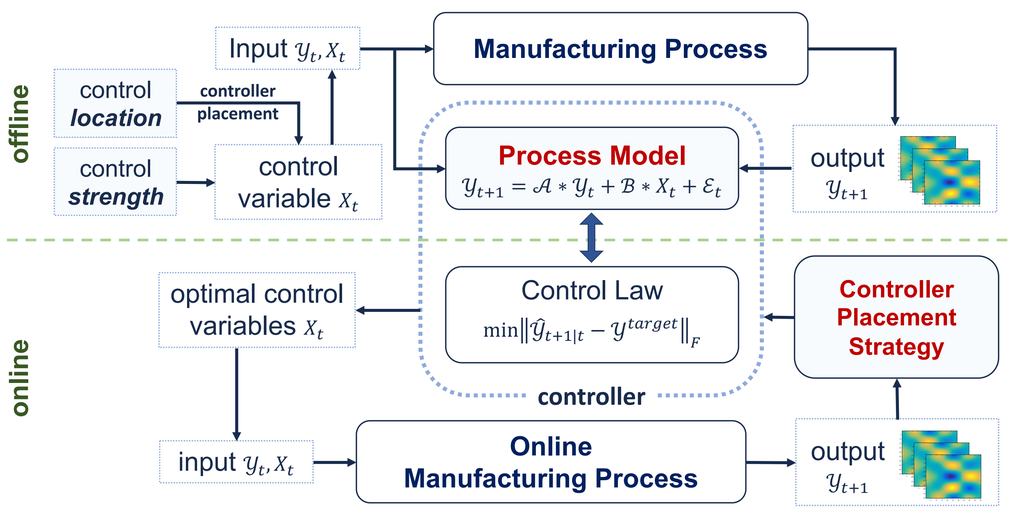
$\mathcal{A}$ models the spatial local correlation between adjacent responses

- $\boldsymbol{c}$: the highest level of the response autocorrelation
- $\boldsymbol{\sigma}$: the unit coverage of the response autocorrelation

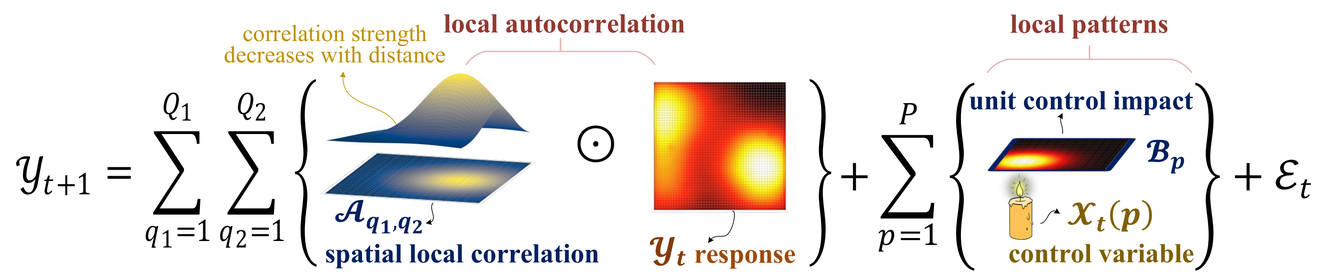
$\mathcal{B}$ models the control impact of location-sensitive control variables

- $\boldsymbol{c}$: the location of control variable
- $\boldsymbol{\sigma}$: the unit control impact in the spatial domain

Methodology Framework



Process Model



$$\mathcal{Y}_{t+1}=\underbrace{\sum_{q_1=1}^{Q_1}\cdots\sum_{q_L=1}^{Q_L}\mathcal{A}_{q_1,\dots,q_L}\odot\mathcal{Y}_t}_{\text{autocorrelated response}}+\sum_{p=1}^{P}\mathcal{B}_p\,\mathbf{X}_t(p)+\mathcal{E}_t$$

control variable noise

Offline Estimation

$$\min_{\boldsymbol{\sigma}_A,\boldsymbol{\sigma}_B}\{\|\mathrm{vec}(\mathcal{Y}_{t+1})-\mathbf{A}\mathrm{vec}(\mathcal{Y}_t)-\mathbf{B}\mathrm{vec}(\mathbf{X}_t)\|_F^2\}$$

subject to $\mathcal{A}_{q_1,\dots,q_L}=\mathcal{K}_{\boldsymbol{c}_{q_1,\dots,q_L};\boldsymbol{\sigma}_{A;q_1,\dots,q_L}}$ and $\mathcal{B}_p=\mathcal{K}_{\boldsymbol{c}_p,\boldsymbol{\sigma}_B}$

Online Control

$$\min_{\widetilde{\mathbf{X}}_t}\left\{\left\|\hat{\mathcal{Y}}_{t+1|t}(\widetilde{\mathbf{X}}_t)-\mathcal{Y}_{t+1}^{\text{target}}\right\|_F^2\right\}$$

subject to $\|\widetilde{\mathbf{X}}_t\|_0=P$ with $\widetilde{\mathbf{X}}_t\in\mathbb{R}^{Q_1\times\dots\times Q_L}$ **Resource Constraint**

$$\hat{\mathcal{Y}}_{t+1|t}(\mathbf{X}_t)=\sum_{q_1=1}^{Q_1}\cdots\sum_{q_L=1}^{Q_L}\hat{\mathcal{A}}_{q_1,\dots,q_L}\odot\mathcal{Y}_t+\sum_{p=1}^{P}\hat{\mathcal{B}}_p\,\mathbf{X}_t(p)$$

Dynamic Controller Placement

$$\min_{\widetilde{\mathbf{X}}_t}\left\{\left\|\hat{\mathcal{Y}}_{t+1|t}(\widetilde{\mathbf{X}}_t)-\underline{\mathcal{Y}_{t+1}^{\text{target}}}\right\|_F^2+\alpha\|\widetilde{\mathbf{X}}_t\|_1\right\}$$

control target

subject to

$$\hat{\mathcal{Y}}_{t+1|t}(\widetilde{\mathbf{X}}_t)=\sum_{q_1=1}^{Q_1}\cdots\sum_{q_L=1}^{Q_L}\underline{\hat{\mathcal{A}}_{q_1,\dots,q_L}}\odot\mathcal{Y}_t+\sum_{q_1=1}^{Q_1}\cdots\sum_{q_L=1}^{Q_L}\underline{\widetilde{\mathcal{B}}_{q_1,\dots,q_L}}\widetilde{\mathbf{X}}_t(q_1,\dots,q_L)$$

offline estimated offline estimated

$$\widetilde{\mathcal{B}}_{q_1,\dots,q_L}=\mathcal{K}_{c_{q_1,\dots,q_L},\hat{\sigma}_B}=\prod_{l=1}^{L}\exp\left(-\frac{1}{2}(\mathcal{D}_l-q_l)^2/\hat{\sigma}_{B;l}^2\right)$$

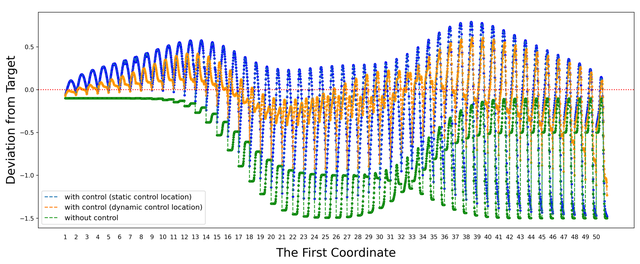
Binary Search Algorithm to determine α to satisfy $\|\widetilde{\mathbf{X}}_t\|_0=P$.

Proposition: The maximum value of α for the binary search is $\alpha_{\max}=\|2\widetilde{\mathbf{B}}^T\mathrm{vec}(\tilde{\mathcal{Y}}_{t+1|t})\|_\infty$, where $\tilde{\mathcal{Y}}_{t+1|t}=\mathcal{Y}_{t+1}^{target}-\sum_{q_1=1}^{Q_1}\cdots\sum_{q_L=1}^{Q_L}\hat{\mathcal{A}}_{q_1,\dots,q_L}\odot\mathcal{Y}_t$.

Numerical Studies

Table I: (2D Case) Relative Mean Square Deviation

RMSD \ P	3	4	5	6
Proposed	0.1251 (2.70e-4)	0.0926 (1.48e-4)	0.0769 (7.28e-5)	0.0685 (6.03e-5)
Static	0.5016 (8.27e-12)	0.3337 (1.11e-11)	0.1539 (8.25e-12)	0.1032 (9.50e-13)



(2D Case) Entry-wise Control Performance

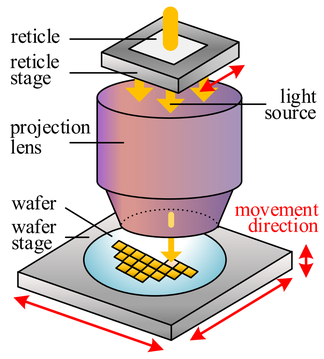
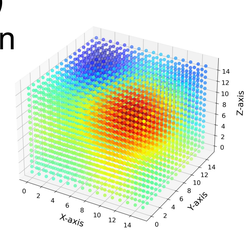
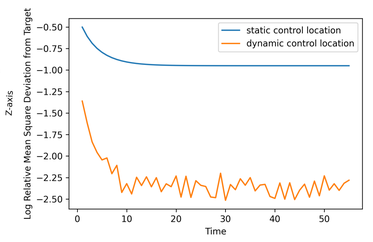


Table II: (Lithography Process) Relative Mean Square Deviation

RMSD \ P	15	30
Proposed	0.2543 (9.34e-8)	0.2524 (1.14e-7)
Static	0.3352 (1.44e-7)	0.2612 (1.17e-7)



Example for 3D Case



(3D Case) Control Performance Over Time

Contributions

- Incorporate (i) spatial local correlation structure within response and (ii) local patterns within the spatial impact of control variables into the system modeling.
- Propose a dynamic control strategy to allocate limited control resources.